Logic Gates Types, Truth Table, Circuit, and Operation

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1 Introduction

Logic gates are the foundation of all digital systems, whether combinational or sequential. A logic gate is a digital circuit with multiple inputs and a single output, and the relationship between the inputs and output follows a certain logic. This logic adheres to the rules of Boolean Algebra.

To develop a deep understanding of logic gates, it is essential to understand the basics of digital signals, binary number systems, and Boolean Algebra.

1.1 Digital Signal

An analog signal is a continuous time-varying current or voltage signal, whereas a digital signal is a pulsating waveform of two discrete values, high and low. These two discrete values are represented by binary numbers 0 and 1. A digital circuit is an electronic circuit that processes digital signals.

- 0 = No, False, Switch Off, Open Circuit, and Low.
- 1 = Yes, True, Switch On, Closed Circuit, and High.

1.2 Binary Numbers

The decimal system has a base or radix of 10, meaning that the number is represented by ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Similarly, the binary system has a base or radix of 2 with only two digits: 0 and 1.

1.3 Boolean Algebra

Boolean Algebra was developed by George Boole, an Irish mathematician. It is a mathematical system using symbols, similar to algebra, for solving binary or logic problems. An equation represented by symbols that follows the laws of Boolean Algebra is known as a Boolean expression.

Three operations used in Boolean Algebra are:

1. OR Operation
2. AND Operation
3. NOT Operation

2 Logic Operations

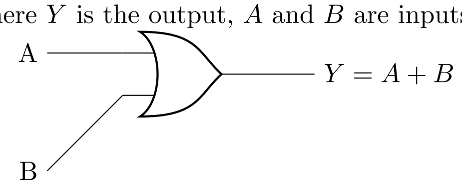
2.1 OR Operation

OR Operation is a form of logic addition represented by the + sign with two or multiple inputs producing one output. The OR Operation produces a HIGH output (1) if at least one of the inputs to the digital circuit is HIGH (1). If both inputs are LOW (0), the output will also be LOW (0).

The Boolean expression for OR Operation is:

$$Y = A + B$$

where Y is the output, A and B are inputs.



A	B	Y = A OR B
0	0	0
0	1	1
1	0	1
1	1	1

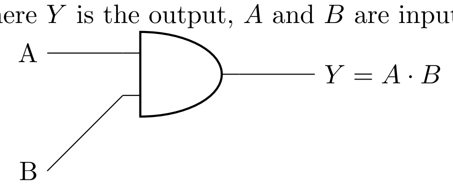
2.2 AND Operation

AND Operation is a form of logic multiplication represented by the ($\cdot$) sign with two or multiple inputs producing a single output. The AND Operation produces a HIGH output (1) only if all inputs to the digital circuit are HIGH (1). If one or both inputs are LOW (0), the output will also be LOW (0).

The Boolean expression for AND Operation is:

$$Y = A \cdot B \quad \text{or} \quad Y = AB$$

where Y is the output, A and B are inputs.



A	B	Y = A AND B
0	0	0
0	1	0
1	0	0
1	1	1

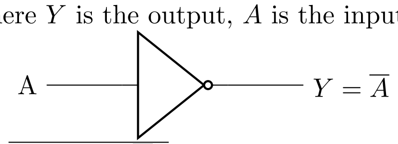
2.3 NOT Operation

NOT is an operation in the form of logic inversion, also known as a complement. There is a single input producing only one output. NOT operation produces a HIGH (1) output for a LOW (0) input and a LOW (0) output for a HIGH (1) input.

The Boolean expression for NOT Operation is:

$$Y = \overline{A}$$

where Y is the output, A is the input.



A	$Y = \overline{A}$
0	1
1	0

3 Logic Gates

A logic gate is a digital circuit with a single output whose value depends upon the logical relationship between the input(s) and output. The relationship between the input values and the output is based on a certain 'logic', hence these circuits are called logic gates.

There are three types of logic gates:

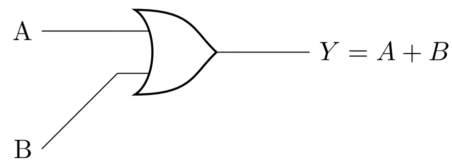
1. **Basic Gates:** OR, AND, and NOT Gates.
2. **Universal Gates:** NAND and NOR Gates.
3. **Derived Gates:** XOR and XNOR Gates.

3.1 Basic Gates

3.1.1 OR Gate

For two or more inputs, the OR gate produces a HIGH (1) output if at least one of the inputs is HIGH (1). The logical expression for an OR Gate is:

$$Y = A + B$$

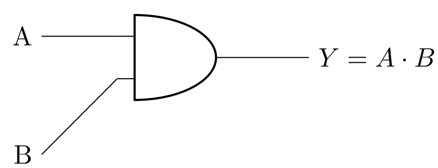


A	B	Y = A OR B
0	0	0
0	1	1
1	0	1
1	1	1

3.1.2 AND Gate

For two or more inputs, the AND gate produces a HIGH (1) output only if all the inputs are HIGH (1). The logical expression for an AND Gate is:

$$Y = A \cdot B \quad \text{or} \quad Y = AB$$

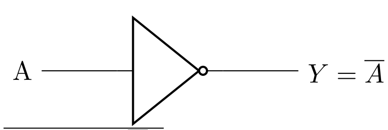


A	B	Y = A AND B
0	0	0
0	1	0
1	0	0
1	1	1

3.1.3 NOT Gate

For a single input, the NOT gate produces the output as a complement of the input. The logic expression for a NOT Gate is:

$$Y = \overline{A}$$



A	$Y = \overline{A}$
0	1
1	0

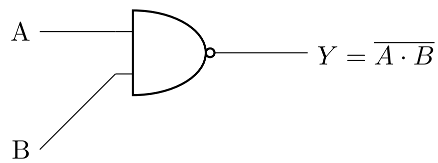
3.2 Universal Logic Gates

3.2.1 NAND Gate

A NAND Gate is a complement of AND Gate. For two or more inputs, the NAND gate produces a HIGH (1) output if at least one of the inputs is LOW

(0). The Boolean expression for a NAND Gate is:

$$Y = \overline{A \cdot B}$$

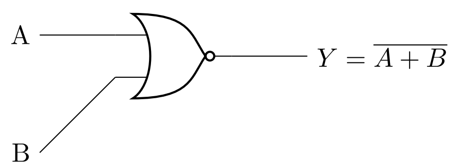


A	B	Y = A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

3.2.2 NOR Gate

A NOR Gate is a complement of OR Gate. For two or more inputs, the NOR gate produces a HIGH (1) output only if all inputs are LOW (0). The Boolean expression for a NOR Gate is:

$$Y = \overline{A + B}$$



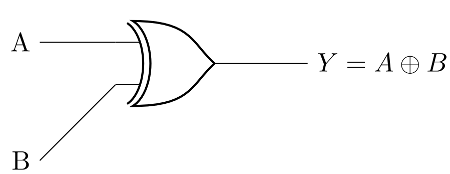
A	B	Y = A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

3.3 Derived Logic Gates

3.3.1 XOR Gate

An XOR (Exclusive OR) gate has two or more inputs but a single output. The XOR gate generates a HIGH (1) output if both inputs are not at the same logic level ($A \neq B$). The Boolean expression for an XOR Gate is:

$$Y = A \oplus B$$

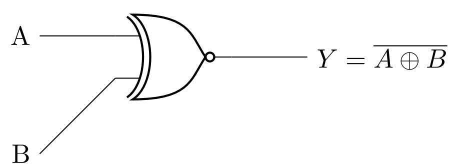


A	B	Y = A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

3.3.2 XNOR Gate

An XNOR (Exclusive NOR) gate has two or more inputs but a single output. The XNOR gate generates a HIGH (1) output if both inputs are at the same logic level ($A = B$). The Boolean expression for an XNOR Gate is:

$$Y = \overline{A \oplus B}$$



A	B	Y = A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

Understanding Potential Dividers in Logic Circuits

What is a Potential Divider?

A potential divider is a basic circuit used to split an input voltage into smaller, adjustable voltages. This is useful in many electronic devices and logic circuits to control voltage levels.

How Does It Work?

A potential divider consists of two resistors connected in series. The input voltage is applied across the entire series combination, and the output voltage is taken from the junction between the resistors.

Diagram

Here's a simple diagram of a potential divider:

In the diagram:

- R_1 and R_2 are the resistors.
- V_{out} is the voltage at the junction between R_1 and R_2.
- V_{in} is the total input voltage.

Expression for Output Voltage

The output voltage V_{out} can be calculated using the formula:

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2} \tag{1}$$

Where:

- V_{in} is the total input voltage.
- R_1 is the resistance of the first resistor.
- R_2 is the resistance of the second resistor.

Example Calculation

If V_{in} is 10V, R_1 is 1kΩ (1000 ohms), and R_2 is 2kΩ (2000 ohms), then:

$$V_{out} = 10\text{V} \times \frac{2000\ \Omega}{1000\ \Omega + 2000\ \Omega} = 10\text{V} \times \frac{2}{3} = 6.67\text{V} \tag{2}$$

This means the output voltage V_{out} will be approximately 6.67V.

Summary

A potential divider is a useful tool for adjusting voltages in circuits. By changing the values of the resistors R_1 and R_2, you can control how much of the input voltage is output.